Journal of Applied Sciences and Nanotechnology

Journal homepage: https://jasn.uotechnology.edu.iq/

Extended Chaotic Nonlinear Programming Technique Constructing with Genetic Algorithms

[1]Bahaa Kamal, *[1]Nadia M. G. Al-Saidi

[1]Department of Applied Science, University of Technology/ Baghdad – Iraq

Article information

Article history:
Received: April, 09, 2021
Accepted: April, 17, 2021
Available online: April, 26, 2021

Keywords:
Non-linear programming,
Evolutionary algorithm,
Chaos theory

Correspondence:
nadia.m.ghanim@uotechnology.edu.iq

Abstract

Chaos theory has attracted much attention because it fully reflects the complexity of the system, which is an essential property in many applications, especially in the optimization problem. In this paper, the possibility of improving research by means of evolutionary algorithms (genetic algorithms) will be discussed which used to solve non- linear programming problems. This improvement and development are carried out using a highly quality chaotic map, which was proposed to be used for generating real values (keys) that are used as reference values for the genetic algorithm. A comparison between the results without using chaotic systems and the results after generating the keys is performed. It shows that the results after the chaotic local search (CLS) are improved and congregate with the optimum value of the solutions obtained by the projected process before the CLS. Moreover, the differences between the proposed systems for improvement are also compared. The evaluation parameters for the proposed chaotic function are developed using the Mathematica 11.2 program.

DOI: 10.53293/jasn.2021.11018, Department of Applied Science, University of Technology

1. Introduction

The problems of non-linear programming are widely recognized in the protective world in many important areas, especially economics problems, engineering design problems, and statistical studies, and most of these problems are of two main types [1, 2]. Constrained optimization problems and unrestricted optimization problems. Until now, there is no clear and general way to solve such problems, since most of these issues are dealt with by the evolutionary algorithm [3] (the genetic algorithm), not always a correct result is performed. It often gives approximate solutions and relatively reliable results, but do not give an optimal solution [4-10]. Recently, various aspects of sciences applied chaos theory, which was defined as the science of unpredictability. It deals with nonlinear phenomena in which their behavior is difficult to predict or control. Chaos theory has attracted much attention because it fully reflects the complexity of the system, which is an essential property in many applications, especially in the optimization problem. In addition to its three main properties (sensitive dependence on initial condition, positive Lyapunov exponent, and stochasticity and ergodicity), this property makes it highly influential to be widely used in many applications, including biology, physics, economics, and philosophy. In addition to these necessary fields, it also shows a high impact in cryptography due to its ability to generate pseudo-random sequences to be used as keys in the encryption process [11-18].

In this effort, this research shall bring these results closer to the correct solution by performing some additions to the genetic algorithm through adding reference points. A newly proposed anarchic function generates those. This will be done by proposing the chaotic function, stating the steps to write it and its advantages, then taking its

results, using them in the genetic algorithm's steps, and extracting the results.
This paper is organized as follows: Section 2 presents some background concepts about chaos theory and evolutionary algorithms. The proposed chaotic function with the steps of the proposed approach is discussed in Section 3. Section 4 outlines the algorithm of the proposed approach. The evaluation aspects of the proposed chaotic function with the key generation are examined in Section 5. Section 6 discussed the results and concluded the paper.

2. Preliminaries

2.1 Evolutionary algorithm (genetic)

The Holland scientist introduced the genetic algorithm in 1970; it is used to find a global or semi-global solution to the proposed problems. This is done by starting with a group of possible solutions. Then several operations applied to them (selection, mutations, and crossover) to create a new generation of chromosomes, which is expected to be of higher quality than the first generation. These operations are repeated many times and the development of generations of chromosomes until they are completed obtaining the best chromosome as an optimal solution.

2.2 Chaos theory

Chaos theory is concerned with studying the behavior of functions subjected to deter- monistic laws and accurate results. They are shown messily, not subject to arrangement and expectation. However, they are highly accurate and influenced by the most specific factors. A wide range in many applications, is presented recently. A messy function is classified into two main types of continuous-time: the primary time factor for such a function in which the results depend on the amount of time that the function takes to operate. The other type is the discrete-time, which depends on the repetition of the results of the map, added it as inputs to the map again, and continues, like this for a large number of iterations; the researcher neutralizes that the number of repetitions is more significant the accuracy of the results is much better.

2.3 Nonlinear programming issues

The general structure of the issue NP is given by the conclusion [2], for an objective function $\varphi(x)$, the inequalities constrains $\Upsilon(x)$, the equalities constrains $\ell(x)$ and the decision vector variables $x = (x_1,\dots, x_n)^T$, we have the optimization problem.

$$
\begin{aligned}
& Min\ \varphi(x) \\
& s.t. \quad \Upsilon i(x) \le 0, \quad i = 1, \dots, p \qquad (1) \\
& \ell j(x) = 0, \quad j = 1, \dots, q \\
& x^l \le x_n \le x^u
\end{aligned}
$$

Where $x_n \in [x^l, x^u]$.

3. The suggested procedure

This topic will discuss the proposed path, which is a merger between the genetic procedure (local research) and the chaos theory. As the basis of the process depends on two main concepts, the first is the gene algorithm, where an approximate solution to the problem is obtained. The second is to improve and bring these solutions to the closest (approximate) correct solution is acceptable using the keys extracted from the chaotic map.

3.1 The first aspect: the genetic algorithm

- **Initial population**

The creation of the primitive generation is an expression that selects a set of inputs that are chosen within a set of conditions and basics upon which the problem to be solved depends.

- **Initial position point**

The system requires at least one visible reference point that is reliable in comparison, transit and development of the next generation of inputs.

- **Repairing**

This part is to fix the inputs that are not targeted in the previous step, amend and make them executable by the achievement in the next iteration. This was done by relying on the initial reference point to be installed.

- **The fitness is evaluated**

In the evaluation stage, fitness for chromosomes is calculated, and its efficiency in crossing the next stage of the algorithm and knowledge of the active chromosome, which approaches the ideal solutions to the problem.

- **The creation of a new generation**

The new generation is formulated by using a set of processes (arrangement, selection, intersection, and mutations).

3.2 The second aspect of the algorithm

The solution using genetic algorithm leads to an optimum approximate solution $x^* = (x_1, x_2,.., x_n)$, and these results are improved and rounded to an optimal and closer solution by using the modification by the chaotic local search (CLS) in the medium of modification of the values of x. The CLS technique is described as follows:

- **Define the scope of chaotic search**

The scope of chaotic search is $[a_i, b_i]$, $i = 1, 2, ..., N$, it is determined by $x_i^* - \varepsilon < a_i$ and $x_i^* + \varepsilon > b_i$, where ε is stated radius of CLS.

- **Creating chaotic Values**

In this stage, different chaotic maps to generate a chaotic random numbers z^k are introduced.

- **Chaos variable maps into the modification range**

Chaos variable z^k is recorded into the modification range of optimization valued $[a_i, b_i]$, such as:

$$x_i^k = [b_i - a_i]z^k + a_i$$
$$= 2\varepsilon z^k - \varepsilon + x_i^*, \quad i = 1, \dots, n \qquad (2)$$

- **Update the best value:**

If $\varphi(x^k) < \varphi(x^*)$ then set $x^k = x^*$, else discontinue the iteration.

- **Stopping the CLS**

If $\varphi(x^*)$ values cannot improve the original results, the chaotic research is stopped and the original values are considered as the closest optimal solution.

4. Communication with Algorithm

The previous steps can be illustrated as follows.

The algorithm

CLS technique; Input data set { $x^* = (x_{1,\dots,} x_n)$, ε, z^0 }	
	While $\varphi(x^*)$ is upgraded
Begin	k=1;
Compute	z^k utilizing the chaotic map in Eq.(2)
If	$\varphi(x^k) < \varphi(x^*) \rightarrow x^* = x^k$
Else,	$\varphi(x^k) \geq \varphi(x^*)$, come to an end; and stop at k=k+1 ;
End	

5. Create keys using the proposed chaotic map

Using the proposed chaotic map the value of the z^k key that will be added to the values of the evolutionary algorithm is created and improved.

5.1 New high chaotic map

A new chaotic function with high messy values was created, and usually this type of function is created by trial and error, the proposed function is described as follows:

$$x_{n+1} = \exp(a{x_n}^2) + \sin(b\,\pi\,x_n) \tag{3}$$

where $a \in [0,1], b \geq 1$. Figure (1) and figure (2) show the approximated solutions for (3) for different values of a and b.

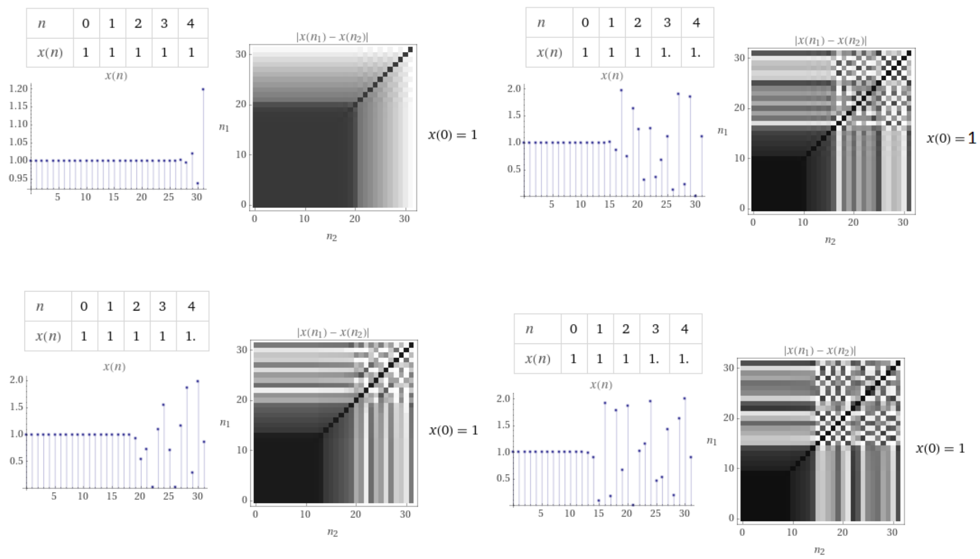

Figure 1. The discrete plotting of Eq.(3) when a=0 and for b=1,…,4 respectively. All values of x_{n+1} approach to 1.

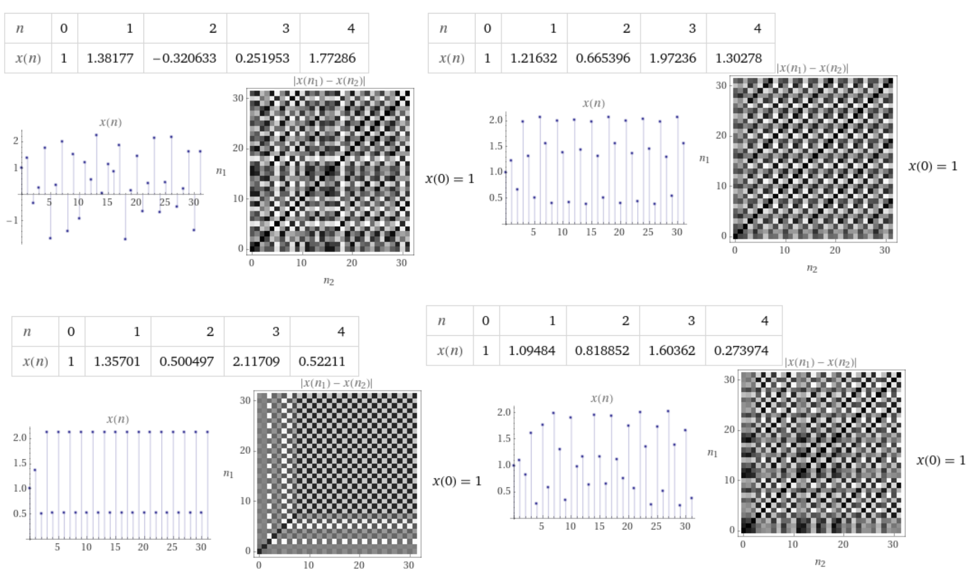

Figure 2. The discrete plotting of Eq.(3) when a=1, 0.5 (first column) 0.25, 0.1 (second column), and for b=1, respectively.

All values of x_{n+1} are decreasing. The chaotic is showed by a set of tests, Lyapunov exponent and bifurcation chaotic behavior appears, and the keys used to improve the genetic algorithm are generated as follows:

- **Lyapunov exponent**

The distinctive sign of Lyapunov for a dynamic system is an amount that un- limitedly separates the rate of nearby tracks. Quantitatively, two space paths vary with the primary separation vector z^k (indicated that the difference can be addressed within linear approximation) at a rate given by

$$|\lambda Z(t)| \approx e^{\lambda t}|\delta Z_0| \tag{4}$$

The separation rate can be varied for different directions of the first separation vector. Thus, there is a spectrum of advocates of Lyapunov equal in number to the dimensional dimensions of the phase. The largest of them is commonly referred to as Maximal Lyapunov exponent (MLE), because it defines the concept of predictability of a dynamic system. Positive MLE is usually taken as an indication that the system is chaotic (provided some other conditions are met, for example, phases space pressure). Note that an arbitrary initial separation vector will usually contain some components in the direction associated with the MLE. So, because of the exponential growth rate, the effect of the other exponent will erase over time. The maximal Lyapunov exponent can be defined as follows:

$$\lambda = \lim_{t\to\infty}\left(\lim_{Z_0\to 0}\frac{1}{t}\right)\ln\left(\frac{|\lambda z|}{|\delta z_0|}\right) \tag{5}$$

Eq.(5) can be viewed in a discrete converge to the formula of the iteration fixed point

$$\lambda = \lim_{n\to\infty}\sum_{j=0}^{n-1}\ln(\varphi'(x_j)) \tag{6}$$

For example, when $\varphi(x_n)=0.5\, x_n^2$, we have $\lambda=\lim_{n\to\infty}\sum_{j=0}^{n-1}\ln((x_j))$ (see figure 3).

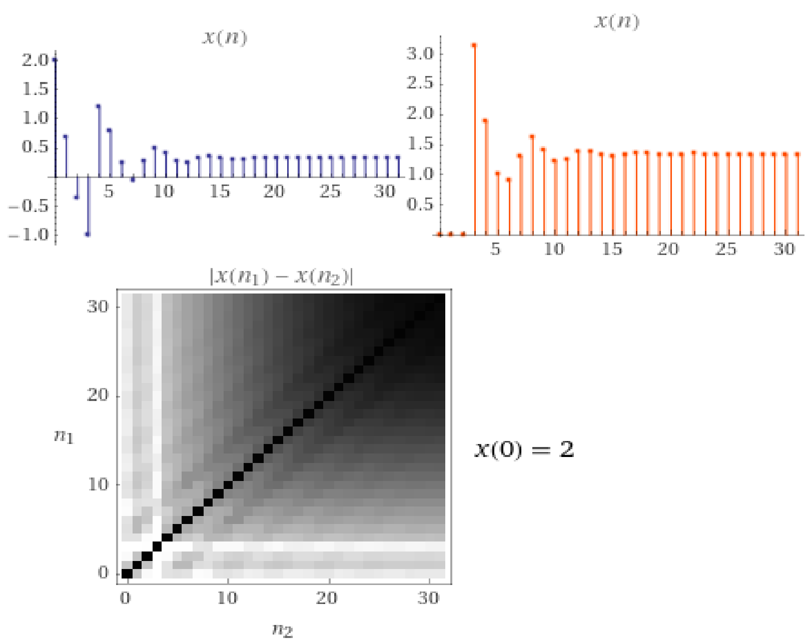

Figure 3. The plot of λ=x(n+1), where the points are the iteration of fixed points

- **Bifurcation**

Local bifurcation occurs when (a) the parameter changes the stability of the equilibrium (or fixed point). In continuous systems, this corresponds to the real portion of the eigenvalue of the equilibrium that runs through zero.

On discrete systems (those described by maps instead of ODEs), this corresponds to a fixed point with a flout multiplier with a coefficient equal to one. In either case, the equilibrium is non-hyperbolic at the bifurcation point. Topological changes in the system phase image can be limited to random small neighborhoods of branched fixed points by moving the bifurcation parameter near the bifurcation point (local). Technically, keep in mind the continuous dynamic system described by the ODE. By using the logistic map, we have (see figure 4, figure 5)

$$x_{n+1} = r\,x_n(1 - x_n) \qquad (n = 0, 1, 2, \ldots) \tag{7}$$

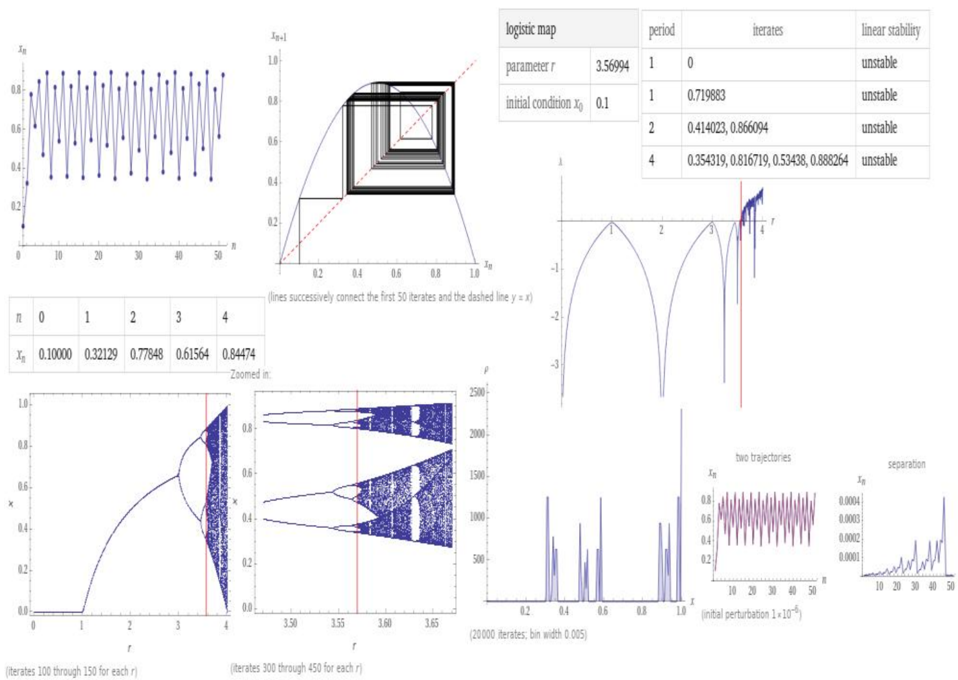

Figure 4. The first row represents the iteration of the Eq. (7) and the limiting behavior, where there is a limit cycle in the iteration 256. The second row indicates the Bifurcation diagram, which is zoomed in the iteration 300 through 450 for the value of r=3.5699. Followed by the Invariant density ρ=0.005, perpetrate trajectories and the last plot is the Lyapunov exponent λ=-0.000797. Therefore, the system is not stable.

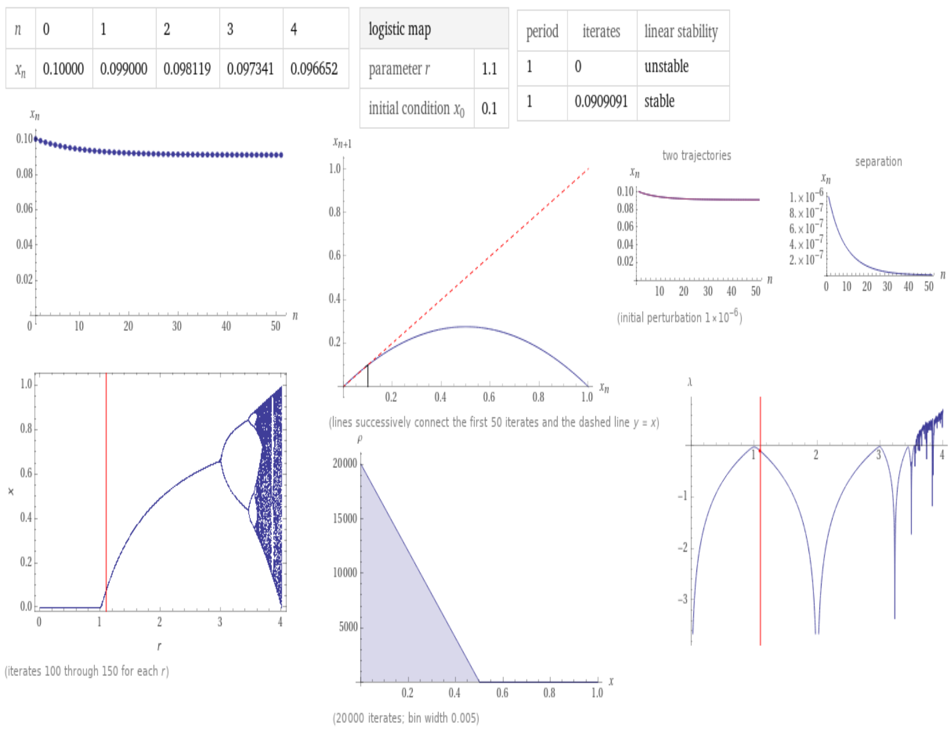

Figure 5. The first row represents the iteration of the fixed points of Eq. (7) while, the second row indicates the Bifurcation diagram, which is zoomed in the iteration 300 through 450 for the value of r=1.1, and x_0=0.1. Followed by the invariant density ρ=0.005, the perpetrate trajectories and the last plot is the Lyapunov exponent λ=-0.105. Therefore, the system is stable after one iteration.

Table 1. Comparison of chaotic genomic algorithm solution and outcome prior to post-CLS.

Optimization method using different chaotic maps	Global solution	The proposed algorithm after Phase I	The proposed algorithm after Phase II
new chaotic map	-450.0000	-449.0942	-450.0000
Chebyshevmap	-443.0100	-439.5730	-450.0000
Singermap	-450.0000	-426.4662	-427.3796
Tentmap	-450.0000	-449.3676	-450.0000
Sine map	-310.0000	-309.9009	-309.9984
Circle map	390.0000	392.1299	391.1724
Piecewise map	-180.0000	-176.3826	-177.3475
Gaussmap	-140.0000	-118.9492	-1 19.9198
Intermittency map	-330.0000	-329.0428	-330.0000
Liebovitch map	-330.0000	-325.5348	-326.0202
Iterative map	90.0000	94.0769	93.276624

6. Discussion and Conclusions

After two applications, two sides, the genetic algorithm and the local chaotic search by applying the messy function. A significant improvement in the results approaching the optimal solutions were achieved. The more the keys are messier; the solutions are closer to the optimal results of the non-linear programming problem. Table 1 shows CGA (chaotic genomic algorithm) (after the first stage and the second stage with the new chaotic map). In the comprehensive solution, from the table, we can see that the planned process results after the CLS are improved and congregate with the optimum value of the solutions obtained by the projected process before the CLS. To check the velocity convergence behavior of the proposed algorithm and provide an arithmetic time comparison, this section is introduced to compare the velocity convergence of the algorithm and the appropriate measurement of the computational time when needed. Businesses are in their inner loops and have altered item dimensions; here, the number of fitness ratings (FEs) has utilized the same way in arithmetic time. In other words, the computation was a measure of the complexity of the algorithm.

References

[1] B.B. Michael. Nonlinear “Optimization with Engineering Applications” Springer Science & Business Media,Vol. 19, 2008.

[2] S.S. Rao. “Engineering Optimization: Theory and practice. 3rdedition”. NY, USA, Wiley; 2009.

[3] H. Derbel, B. Jarboui, S. Hanafi, H. Chabchoub. “Genetic Algorithm with Iterated Local Search for Solving Allocation Routing Problem.” Expert Systems with Applications 39.3,2865-2871, 2012.

[4] S. Arora, H. Singh, M. Sharma, S. Sharma, and P. Anand. "A New Hybrid Algorithm based on Grey Wolf Optimization and Crow Search Algorithm for Unconstrained Function Optimization and Feature Selection.” IEEE Access, 7, 26343-26361, 2019.

[5] M.A. El-Shorbagy, A.A. Mousa, and S.M. Nasr. "A Chaos-based Evolutionary Algorithm for General Nonlinear Programming Problems." Chaos, Solitons & Fractals 85 (2016): 8-21.

[6] T. Singh. "A Chaotic Sequence-guided Harris hawks Optimizer for Data Clustering." Neural Computing and Applications, 32, 17789-17803,2020.

[7] M.A. Elsisy, D.A. Hammad, and M.A. El-Shorbagy. "Solving Interval Quadratic Programming Problems by using the Numerical Method and Swarm Algorithms." Complexity, 2020.

[8] Demidova, A. Liliya, and A. V. Gorchakov. "A Study of Chaotic Maps Producing Symmetric Distributions in the Fish School Search Optimization Algorithm with Exponential Step Decay." Symmetry 12(5), 784, 2020.

[9] G. Debolina, and J. Singh. "Spectrum-based Multi-fault Localization using Chaotic Genetic Algorithm." Information and Software Technology, 106512, 2021.

[10] N. Mobaraki, R. Boostani, and M. Sabeti. "Chaotic-based Particle Swarm Optimization with Inertia Weight for Optimization Tasks." Journal of AI and Data Mining 8, no. 3, 303-312, 2020.

[11] N.M.G. Al-Saidi, D. Younus, H. Natiq, M.R.K. Ariffin, M.A. Asbullah, Z. Mahad. “A New Hyperchaotic Map for a Secure Communication Scheme with an Experimental Realization.” Symmetry. 12(11), 1881, 2020.

[12] N.A. Alwan, A.Y. Yousif, N.M.G. Al-Saidi. “Performance-enhancing of RSA Public Key via Three-Dimensional Hyperchaotic System.” AIP Conference Proceedings. 2325(1), 020027, 2021

[13] M.S. Fadhil, A.K. Farhan, M.d.N. Fadhil, N.M.G. Al-Saidi. “A New Lightweight AES using a Combination of Chaotic Systems.” IEEE Conference on Information Technology to Enhance E-learning and Other Application.” IT-ELA, 82-88, 2020.

[14] D.S. Ali, N.A. Alwan, N.M.G. Al-Saidi. “Image Encryption based on Highly Sensitive Chaotic System.” AIP Conference Proceedings 2183, 080007,2019.

[15] A. K. Farhan, N.M.G. Al-Saidi, A.T. Maolood, F. Nazarimehr, I. Hussain. “Entropy Analysis and Image Encryption Application based on a New Chaotic System Crossing a Cylinder.” Entropy 21(10), 958, 2019.

[16] A.K. Farhan, R.S. Ali, H. Natiq, N.M.G. Al-Saidi. “A New S-Box Generation Algorithm based on Multistability Behavior of a Plasma Perturbation Model.” IEEE Access 7,124914-124924, 2019.

[17] H. Natiq, N.M.G. Al-Saidi, M.R.M. Said, A. Kilicman. “A New Hyperchaotic Map and its Application for Image Encryption.” The European Physical Journal Plus 133 (1), 1-14, 2018.

[18] V.T. Pham, D.S. Ali, N.M.G. Al-Saidi, K. Rajagopal, F.E. Alsaadi, S. Jafari. “A Novel Mega-stable Chaotic Circuit.” Radioengineering Journal. 29(1), 140-146, 2020.