Journal of Applied Sciences and Nanotechnology

Journal homepage: https://jasn.uotechnology.edu.iq/

# Calculating Modern Roman Domination of Fan Graph and Double Fan Graph

**[1]Saba Salah*, [2]Ahmed A. Omran, [1]Manal. N. Al-Harere**

[1]Department of Applied Sciences, University of Technology – Iraq

[2]Department of Mathematics, College of Education for Pure Sciences, University of Babylon – Iraq

**Article information**

***Article history:***
Received: June, 28, 2021
Accepted: October, 09, 2021
Available online: June, 10, 2022

***Keywords*:**
Modern Roman domination,
Fan Graph,
Corona,
Roman dominating set

***Corresponding Author:***
Saba Salah
as.18.95@grad.uotechnol.edu.iq

**Abstract**

This paper is concerned with the concept of modern Roman domination in graphs. A Modern Roman dominating function on a graph is labeling such that every vertex with label 0 is adjacent to two vertices; one of them of label 2 and the other of label 3 and every vertex with label 1 is adjacent to a vertex with label 2 or label 3. The weight of a Roman dominating function is the value $f(V) = \sum_{v \in V} f(v)$. The minimum weight of all possible Roman dominating functions is called the "Roman Domination Number" of a graph. This dominance can be used in many aspects of life, for example in computer networks, transmission lines, and many others. In this paper, the modern Roman domination of the fan graph and the double fan graph with their complement are determined. Also, it has been determined the the number of modern Roman dominations of the corona of two specific graphs like the corone of two fan graph, two double fan graph ,fan graph and double fan graph and the oppisit of them.

DOI: 10.53293/jasn.2021.3906.1060, Department of Applied Sciences, University of Technology


## 1. Introduction and Basic Concepts

A graph is a nonempty set whose elements are called vertices or points. It also contains a set of elements consisting of unordered pairs of vertices; these elements are called edges or lines. For theoretic terminology and the basic concept of a graph see [1]. There are many relations between graph theory and other branches of mathematics such as Topology, Algebra, Probability, Fuzzy and Numerical Analysis. In addition, there are relations with other sciences such as Engineering, Computer Science, Chemistry, Physics, and Biology. The concept of graph domination is one of the topics in graph theory, in which it is used in all the above sciences. The first one who initiated this concept is Claude Berge in 1962 [2]. Ore [3] is the one who introduced the concepts of domination number and dominating sets. After that, this notion started to appear in different kinds and forms. In mathematics, this concept appeared in many fields including fuzzy graph [4-6] topological graph [7], polynomials domination [8, 9], and others [10, 11]. Additionally, many new definitions in this concept have been used, depending on putting some conditions on the dominating set [12],[13-16], out of dominating set [17-19] or on the two together as in [13]. A Roman dominating function on a graph $G = (V; E)$ is a function $f: V(G) \to \{0,1,2\}$satisfying the condition that every vertex $u$ for which $f(u) = 0$ is adjacent to at least one vertex $v$ for which $f(v) = 2$, [20, 21].

In this paper, the definition of modern Roman domination in [22] is used by labeling the function on fan graph (the fan $\boldsymbol{F_n}$ defined to be the graph $\boldsymbol{p_n}+\boldsymbol{K_1}$ ) and double fan graph (the double fan $\boldsymbol{DF_n}$defined to be the graph $\boldsymbol{p_n} + \overline{\boldsymbol{k_2}}$ ), [1].

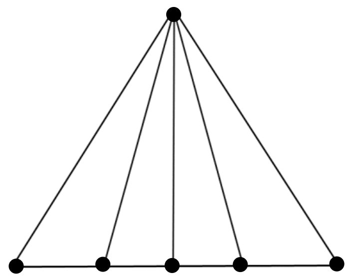

**Figure 1:** Fan graph of order 5.

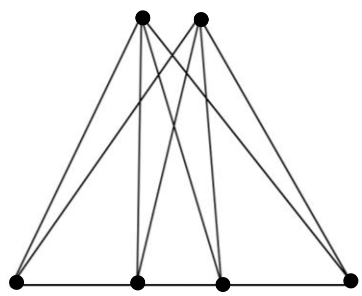

**Figure 2:** Double fan graph of order 4.

**Definition 1.1.** [22] Let $f: V(G) \to \{0,1,2,3\}$ be a labeling function where $G$ is a graph such that every vertex with label 0 is adjacent to two vertices, one of them of label 2 and the other of label 3, and every vertex with label 1 is adjacent to a vertex with label 2 or label 3, so this function is called a modern Roman dominating function.

The modern Roman domination number $\gamma_{mR}(G)$ of G is the minimum $f(V) = \sum_{v \in V} f(v)$ over all such functions of G. The minimum weight of all these functions is called the modern Roman domination number and is denoted by $\gamma_{mR}(G)$. (as an example see Fig. 3)

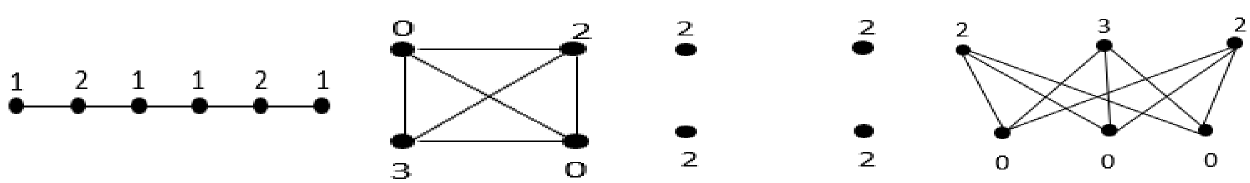


**Figure 3:** Modern Roman domination.

**Proposition 1.2.** [22] If G is a graph of order n and has a modern Roman domination $\gamma_{mR}(G)$, then
1) If $n \geq 4$, then $5 \leq \gamma_{mR}(G) \leq 2n$.
2) If there are two vertices adjacent to all other vertices in G, then $\gamma_{mR}(G) = 5$.
3) If Gis null, then $2\gamma = \gamma_{mR}$.
4) $V_2 \neq \emptyset$.
5) $V_2 \cup V_3$ is a dominating set of induced subgraph $G[V_0]$.
6) If v is a pendant vertex, then $f(v) \neq 0$.
7) If v is an isolated vertex, then $f(v) = 2$.

**Theorem 1.3.** [22] The modern Roman domination of $P_n$ is

$$\gamma_{mR}(P_n) = n + \lceil \frac{n}{3} \rceil.$$

**Proposition 1.4.** [23] Let $P_n$ be a Path graph of order n The modern Roman domination of $\overline{(P_n)}$ is

$$\gamma_{mR}(\overline{P_n}) = \begin{cases} 2n, if n = 1,2 \\ 5, if n = 3 \\ 6, if n \geq 4 \end{cases}.$$

**Proposition 1.5.** [22] For $n \geq 1$, $\gamma_{mR}(K_n) = \begin{cases} n+1, if n \leq 3 \\ 5, if n \geq 4 \end{cases}$.

## 2. Calculating Modern Roman domination of fan graph and double fan graph

**Theorem 2.1.** Let $F_n$ be a fan graph of order n, then the modern Roman domination of $F_n$ is

$$\gamma_{mR}(F_n) = \begin{cases} n+2, if n = 1,2 \\ 2\lfloor \frac{n}{3} \rfloor + 3, if n \equiv 0(mod 3) \\ 2\lfloor \frac{n}{3} \rfloor + 4, if n \equiv 1(mod 3), n \neq 1 \\ 2\lceil \frac{n}{3} \rceil + 3, if n \equiv 2(mod 3), n \neq 2 \end{cases}$$

Proof. It is known that the fan graph $F_n$ is a join of two graphs $P_n$+$K_1$ , there are two different cases to label as follows;

Case 1. If $n = 1,2$, then Let $v_1$ be the vertex that represents the graph $K_1$ in which the vertices of the path of order 1,2 are $v_2$, $v_3$. If the vertex $v_1$ is labeled by 2 and the other vertices are adjacent to $v_1$, so the minimum label value can label the vertices of $p_n$by 1, $f(v_2) = 1$,$f(v_3) = 1$.Then the modern Roman domination of is $n + 2$.

Case 2. If $n \neq 1,2$, then let $v_1$ be the vertex that represents the graph $K_1$ and the vertices of the path of order $n$ are $v_2$, $v_3$, $v_4$, …, $v_n$ . The vertex $v_1$ is labeled by 3 and the other vertices are adjacent to $v_1$. There are three different subcases to label it as follows.

Subcase 1. If $n \equiv 0(mod3)$, then $f(v_2) = 0$, $f(v_3) = 2$, $f(v_4) = 0$, and so on for every consecutive three vertices. Thus, $f(v_n) = 0$ and $f(v_{n-1}) = 2$, which means there is no problem with these labels, therefore, $\gamma_{mR}(F_n) = 2\lfloor\frac{n}{3}\rfloor + 3$.

Subcase 2. If $n \equiv 1(mod3)$, then let $v_2 = 0, v_3 = 2, v_4 = 0$ and so on for every consecutive three vertices. It is obvious that $n - 1 \equiv 0(mod3)$, then the first $(n - 1)$ vertices have labeled as in subcase 1. The remaining vertex is $v_n$, this vertex cannot take 0 or 2 labels, since the vertex $v_{n-1}$ takes zero. Thus, the minimum label to the vertex $v_n$ is one. Therefore, $\gamma_{mR}(F_n) = 2\lfloor\frac{n}{3}\rfloor + 4$.

Subcase 3. If $n \equiv 2(mod3)$, then in the same manner in subcase 1, the first three vertices have been labeled as in subcase 1, and so on for every consecutive three vertices, since $n - 2 \equiv 0(mod3)$. Thus, for these values of labels, the vertex $v_{n-1}$ takes zero and the vertex $v_n$ takes two. Therefore, $\gamma_{mR}(F_n) = 2\lceil\frac{n}{3}\rceil + 3$.

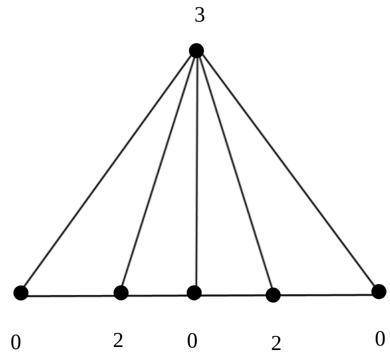


**Figure 4:** Modern Roman domination of fan graph.

**Proposition 2.2.** Let $\mathrm{DF_n}$ be a double fan graph of order n, then the modern Roman domination of $\mathrm{DF_n}$is

$$\gamma_{mR}(DF_n) = \gamma_{mR}\left(P_n + \overline{k_2}\right) = \begin{Bmatrix} 4, if n = 1 \\ 5, if n \geq 2 \end{Bmatrix}$$

Proof. It is known that the double fan graph $DF_n$ is a join of two graphs$P_n + \overline{k_2}$. Let $v_1$ , $v_2$ be the vertices that represent the graph $\overline{k_2}$ and the vertices of the path of order $n$ are $v_3$, $v_4$, …, $v_n$. There are two different cases to label as follows;

Case 1. If $n = 1$ then the vertices $v_1$ and $v_2$ are labeled by 1, and the vertex $v_3$ by 2. So, the modern Roman domination of double fan graph is equal to 4, if $n = 1$.

Case 2. If $n \geq 2$ then the vertex $v_1$ is labeled by 2, and the vertex $v_2$ by 3, and the other vertices are adjacent to $v_1$ and $v_2$, so the minimum label value can label the vertices of $p_n$ by 0,$f(v_3) = 0$, $f(v_4) = 0$, $f(v_5) = 0$, and $f(v_6) = 0$ and so on. Then if there are two vertices adjacent to all other vertices in $G$, then $\gamma_{mR}(G) = 5$. So, the modern Roman domination of double fan graph is equal to 5 if $n \geq 2$.

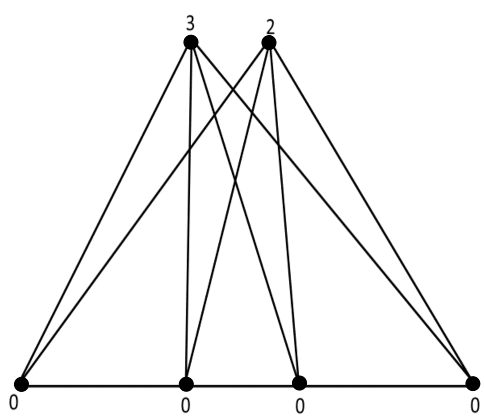


**Figure 5:** Modern Roman domination of double fan graph.

## 3. Calculating Modern Roman Domination on the Complement of Fan Graph and Double Fan Graph

**Proposition 3.1.** Let $\mathrm{F_n}$ be a fan graph of order n, then the modern Roman domination on a complement of fan graph $\overline{\mathrm{F_n}}$is

$$\gamma_{mR}(\overline{F_n}) = \gamma_{mR}(\overline{P_n}) + 2 = \begin{cases} 2n+2, if n = 1, 2 \\ 7, if n = 3 \\ 8, if n \geq 4 \end{cases}$$

Proof. It is clear that $\overline{F_n} \equiv \overline{P_n} \cup K_1$, so the result is straightforward from Proposition 1.4 and Proposition 1.5.

**Proposition 3.2.** Let $DF_n$ be a double fan graph of order n, then the modern Roman domination on a complement of double fan graph $\overline{DF_n}$ is

$$\gamma_{mR}(\overline{DF_n}) = \gamma_{mR}(\overline{P_n}) + 3 = \begin{cases} 2n+3, if n = 1, 2 \\ 8, if n = 3 \\ 9, if n \geq 4 \end{cases}$$

Proof. It is clear that $\overline{DF_n} \equiv \overline{P_n} \cup K_2$, so the result is straightforward from Proposition 1.4 and Proposition 1.5.

**4. Calculating Modern Roman Domination on the Corona of Two Graphs**

**Proposition 4.1.** The modern Roman domination of corona of two fan graphs $F_n$ and $F_m$ is

$$\gamma_{mR}(F_n \odot F_m) = \begin{cases} 4(n+1), if m = 1 \\ 5(n+1), if m \neq 1 \end{cases}$$

Proof. There are two different cases to label as follows;

Case 1. If $m = 1$ then the vertices of $F_n$ are labeled by 2, and the vertices of $F_m$ by 1. So, the modern Roman domination of corona of two fan graphs is equal to $4(n+1)$ if $m = 1$.

Case 2. If $m \neq 1$ then the vertices of $F_n$ are labeled by 3 and Let $v_1$ be the vertex that represents the graph $K_1$ of $F_m$. Then $v_1$ is labeled by 2, and other vertices of $F_m$ are labeled by 0 from Proposition 1.2. So, the modern Roman domination of corona of two fan graphs is equal to $5(n+1)$ if $m \neq 1$.

**Proposition 4.2**. The modern Roman domination of corona of fan graph $F_n$ and complement fan graph $\overline{F_m}$ is

$$\gamma_{mR}(F_n \odot \overline{F_m}) = \begin{cases} 4(n+1), if m = 1 \\ 5(n+1), if m = 2 \\ 7(n+1), if m \geq 3 \end{cases}$$

Proof. There are three different cases to label as follows;

Case 1. If $m = 1$ then the vertices of $F_n$ are labeled by 2, and the vertices of $\overline{F_m}$ are labeled by 1. So, the modern Roman domination of corona of fan graph $F_n$ and complement fan graph $\overline{F_m}$ equals $4(n+1)$ if $m = 1$.

Case 2. If $m = 2$, then the vertices of $F_n$ are labeled by 2 and the vertices of $\overline{F_m}$ by 1. So, the modern Roman domination of corona of fan graph $F_n$ and complement fan graph $\overline{F_m}$ equals $5(n+1)$ if $m = 2$.

Case 3. If $m > 3$, then the vertices of $F_n$ are labeled by 2 and let $v_1$ be the vertex that represents the graph $K_1$ of $\overline{F_m}$ and the vertices of the complement path of order n are $v_2, v_3, v_4, \dots, v_n$. the vertices $v_1, v_2, v_3$ are labeled by 1,3,1 respectively. and the other vertices of $\overline{F_m}$ are labeled by 0 from Proposition 1.2. So, The modern Roman domination of corona of fan graph $F_n$ and complement fan graph $\overline{F_m}$ equals $7(n+1)$ if $m \geq 3$.

**Proposition 4.3.** The modern Roman domination of corona of two double fan graph $DF_n$ and $DF_m$ graphs is

$$\gamma_{mR}(DF_n \odot DF_m) = 5(n+2), \text{ Where } n \geq 1, m \geq 1$$

Proof. Let $v_1$ , $v_2$ be the vertices, which represent the graph $\overline{k_2}$ of graph $DF_m$, then the vertex $v_1$ is labeled by 2 and the vertex $v_2$ by 3 and the other vertices are labeled by 0 according to Proposition 1.2. The modern Roman domination of corona of two double fan graph $DF_n$ and $DF_m$ graphs is equal to $5(n+2)$ , where $\geq 1, m \geq 1$.

**Proposition 4.4.** The modern Roman domination of corona of double fan graph $\mathrm{DF_n}$ and complement double fan $\overline{\mathrm{DF_m}}$ graphs is

$$\boldsymbol{\gamma_{mR}\left(DF_n \odot \overline{DF_m}\right) = \begin{cases} (5+m-1)(n+2), if m = 1,2,3 \\ 8(n+2), if m > 3 \end{cases}}$$

Proof. There are two different cases to label as follows;
Case 1. If $m = 1,2,3$ then the vertices of $DF_n$ are labeled by 2 and the vertices of $\overline{DF_m}$ by 1. So, the modern Roman domination of corona of double fan graph $DF_n$ and complement double fan graph $\overline{DF_m}$ is equal to $(5+m-1)(n+2)$.

Case 2. If $m > 3$ then the vertices of $DF_n$ are labeled by 2, and let $v_1, v_2$ be the vertices, which represent the graph $K_2$ of $\overline{DF_m}$ and the vertices of the complement of the path of order $m$ are $v_3, v_4, \ldots, v_n$. Thus, the vertices $v_1, v_2$, $v_4$ are labeled by 1. The vertex $v_3$ is labeled by 3 and the other vertices of $\overline{DF_m}$ are labeled by 0 from Proposition 1.2. So, the modern Roman domination of corona of Fan $DF_n$ and complement Fan $\overline{DF_m}$ graphs, equals $8(n+2)$ if $m > 3$.

**Proposition 4.5.** The modern Roman domination of corona of fan $\mathrm{F_n}$ and double fan $\mathrm{DF_m}$ graphs is

$$\boldsymbol{\gamma_{mR}(F_n \odot DF_m) = 5(n+1)}$$

Proof. Let $v_1, v_2$ be the vertices, which represent the graph $\overline{k_2}$ of $DF_m$. The vertices $v_1, v_2$ are labeled by 2 and 3, then the other vertices are labeled by 0 from Proposition 1.2. So, the modern Roman domination of corona of fan $F_n$ and double fan $DF_m$ graphs, is equal to $5(n+1)$.

**Proposition 4.6.** *The modern Roman domination of corona of double fan $DF_n$ and fan $F_m$ graphs is*

$$\boldsymbol{\gamma_{mR}(DF_n \odot F_m) = \begin{cases} 4(n+2), if m = 1 \\ 5(n+2), if m \neq 1 \end{cases}}$$

Proof. There are two different cases to label as follows;
Case 1. If $m = 1$ then let $v_1$ be the vertex, which represents the graph $K_1$ of $F_m$. The vertex $v_1$ is labeled by 2 and the other vertices which are adjacent to $v_1$ are labeled by 1, so the modern Roman domination of corona of double fan $DF_n$ and fan $F_m$ graphs is equal to $4(n+2), if m = 1$.

Case 2. If $m \neq 1$ then the vertices of $DF_n$ are labeled by 2. Let $v_1$ be the vertex, which represents the graph $K_1$ of $F_m$. The vertex $v_1$ is labeled by 3, and the other vertices are labeled by 0 according to Proposition 1.2. Therefore, the modern Roman domination of corona of double fan $DF_n$ and fan $F_m$ graph is equal to $5(n+2), if\ m \neq 1$.

## 5. Applications about modern Roman domination

A new model of graph domination is introduced, based on the Roman domination function that called “modern Roman domination” (MRDF). This definition will identify the ways of defense in war zones with four weapon types; a light weapon for pedestrians, medium weapons, and heavy weapons such as tanks, missiles and finally the fourth weapon which was the air force. The applied conditions for this defense strategy success were : a light weapon that is supported by heavy weapons and air force coverage. While,the medium weapon is supported with a heavy weapon or air force coverage.

The defense strategy of modern Roman domination is based on the fact that every place in which there is established a modern Roman legion (labels 2 and 3 in the modern Roman dominating function) is able to protect themselves under external attacks; and that every unsecured place (labels 0 and 1) such that (labels 0) must have stronger neighbors (label 2 and 3), while (labels 1) must have a stronger neighbor (label 2 or 3). In that way, if an unsecured place (a label 0) is attacked, then the stronger neighbors could send the two legions in order to defend the weak neighbor vertex (label 0) from the attack.

## 6. Conclusions

In this work, it has been determined the modern Roman domination number of both fan graph and double fan graph. In addition, it had been calculated the modern Roman domination on the complement of fan graph, double fan graph, and modern Roman domination on the corona of two graphs.

## Conflict of Interest

We have no conflict of interest.

## References

[1] F. Harary, “Graph Theory ”, Reading Mass, Addison Wesley ,1969.

[2] C. Berge, “ The theory of graphs and its applications”, Methuen and Co, London,1962.

[3] O. Ore, “ Theory of Graphs,” American Mathematical Society, Providence, R.I. ,1962.

[4] Sahib Sh. Kahat, A. A. Omran, and M. N. Al-Harere, “ Fuzzy equality co-neighborhood domination in graphs,” *International Journal of Nonlinear Analysis and Applications*, vol.12 ,No.2, p.537-545,2021.

[5] A. A. Omran and T. A. Ibrahim, “ Fuzzy co-even domination of strong fuzzy graphs,” *International Journal of Nonlinear Analysis and Applications*, vol.12, No. 1, p.727-734,2021.

[6] H. J. Yousif and A. A. Omran, “ 2-anti fuzzy domination in anti-fuzzy graphs,” *IOP Conference Series: Materials Science and Engineering,* vol.928, p.1-11, 2020.

[7] A. A. Jabor and A. A. Omran, “ Hausdorff Topological of Path in Graph,” *IOP Conference Series: Materials Science and Engineering*,vol. 928,p.1-11 2020.

[8] I. A. Alwan and A. A. Omran, “ Domination polynomial of the composition of complete graph and star Graph,” *Journal of Physics Conference Series*,vol.1591,p.1-7, 2020.

[9] M. N. Al-Harere, “ The primary decomposition of the factor group $cf(G,Z)/\overline{R}$(G), *” Engineering and Technology Journal*,vol.29, No.5, p.1-8,2011.

[10] A. A. Omran, “ Domination and Independence on Square Chessboard,” *Engineering and Technology Journal*,vol. 35, No.1,p.1-8,2017.

[11] M. A. Abbood, A. A. AL-Swidi and A. A. Omran, "Study of Some Graphs Types via. Soft Graph," *Journal of Engineering and Applied Sciences*,vol.14, p. 10375-10379,2019.

[12] M. N. Al-Harere and M. A. Abdlhusein, “ Pitchfork domination in graphs,” *Discrete Mathematics, Algorithm and Applications*, vol.12,No.02, p. 2050025-13,2020.

[13] M.N. Al-Harere, A. A. Omran, and A. T. Breesam, “ Captive domination in graphs,” *Discrete Mathematics, Algorithms and Applications*, vol.12, No.06, p.2050076-10,2020.

[14] M. N. Al-Harere, R. J. Mitlf and F. A. Sadiq,“ Variant domination types for a complete h-ary tree,” *Baghdad Science Journal*,vol.18, No.1, p. 2078–8665,2021.

[15] M. N. Al-Harere, P. A. Khuda,“Tadpole Domination in duplicated graphs”,*Discrete Mathematics, Algorithms and Applications*, vol.13, No.02, p.2150003,2021.

[16] M. A. Abdlhusein and M. N. Al-Harere,“Total pitchfork domination and its inverse in graphs,” *Discrete Mathematics, Algorithm and Applications*, vol.13, No.04, p.2150038,2021.

[17] M. A. Abdlhusein and M. N. Al-Harere, “New parameter of inverse domination in graphs,” *Indian Journal of Pure and Applied Mathematics*, vol.25, No.1, p.281-288,2021.

[18] Sahib Sh. Kahat and M. N. Al-Harere,“Inverse equality co-neighborhood domination in graphs,”*Journal of Physics: Conference Series*,vol.1879,p.032036,2021.

[19] A. A. Omran and M. M. Shalaan,“Inverse co-even domination of graphs,”*IOP Conference Series: Materials Science and Engineering*,vol.928, p.042025-6, 2020.

[20] E.J. Cockayne, P.M. Dreyer Jr, S.M. Hedetniemi et al, “ On Roman domination in graphs," *Discrete Mathematics,* vol.278 , p.11-22,2004.

[21] G. J. Chang, S. H. Chen, and C. H. Liu, “ Edge Roman domination on graphs,” *Graphs and Combinatorics*,vol.32, p.1731–1747,2016.
[22] A. A. Omran and H. J. Al Hwaeer,“ Modern Roman Domination in graphs," *Basrah Journal of Science (A)*,vol.36, p .45-54,2018.
[23] S. Salah, A. A. Omran and M. N. Al-Harere, “ Modern Roman Domination on two operations in certain graphs,*” American Institute of Physics Journal*,under publication.