Journal of Applied Sciences and Nanotechnology, Vol. 5, No. 1 (2025)

Journal of Applied Sciences and Nanotechnology

Journal homepage: jasn.uotechnology.edu.iq

A Mathematical Model of the Electrical Power Management Systems Using the Core Values in Cooperative Game

Mohammed Musa, Jabbar Abbas*

Department of Applied Sciences, University of Technology - Iraq, Iraq

A R T I C L E I N F O

Article history:
Received: September, 07, 2024
Accepted: March, 03, 2025
Available online: March, 10, 2025

Keywords:
Optimization problem,
Cooperative game,
Electrical power systems,
Core value

Corresponding Author:
Jabbar Abbas
Jabbar.a.ghafil@uotechnology.edu.iq

A B S T R A C T

In the context of game theory, cooperative game has been applied in several fields and can be successfully used to evaluate the players (people or companies) involved. In cooperative game theory, the core is a concept that represents the set of feasible allocations (or distributions of total payoff) among players that cannot be improved upon by any coalition of players. This paper aims to apply a mathematical model to modeling cooperation among power stations and fuel supply producers using a core value-based optimization algorithm. We use the cooperative game to show the potential cost in cooperation through an optimization algorithm to find the most feasible solution using the Python program as a working procedure. Then, we apply the working method to the case of fuel supply and electricity generation in Wasit Thermal Power Plant in cooperation. The outcomes of the proposed methodology will greatly help professionals to formulate and improve well-structured strategies for future electrical energy systems in the Wasit Thermal Power Plant.

https://doi.org/10.53293/jasn.2025.7507.1322, Department of Applied Sciences, University of Technology - Iraq.

1. Introduction

The various industrial developments in electrical energy based on different types of electrical energy give great importance to the electrical energy sector. Hence, there is an increasing interest in using scientific methods, developing productive capabilities to make decisions and improve well-structured strategies for future electrical energy systems. For this purpose, many research studies investigate different optimization techniques of electrical power systems [1-4]. Although electrical power optimization is widely regarded as the epitome of modernity, there is growing skepticism among individuals in nations that have adopted these programs, particularly in locations such as Iraq, over their actual efficacy. It has been shown that the expense of optimizing energy production can be substantial compared to the advantages and profits it brings.

In the context of game theory, cooperative game has been applied in the several fields [5-13] and can be successfully used to evaluate the players (people or companies) involved. The concept of interactions among players has been applied to several problems in the decision theory [14-21]. In cooperative game theory, the core [22] is a concept that represents the set of feasible allocations (or distributions of total payoff) among players that cannot be improved upon by any coalition of players. In other words, an allocation is in the core if no subset of players (a coalition) can form their group and achieve a better payoff than what they receive in the allocation.

In this study, we apply a mathematical model to modeling cooperation among power stations and fuel supply producers using the core in the cooperative games to evaluate the costs between power plants and oil producers in oil fields through optimization algorithm and Python program as a working procedure to find the most feasible solution. Then we apply a working method to study the case of fuel supply and electricity production in the Wassit* thermal station in case of cooperation.

The outline of his paper is organized as follows. Section 2 introduces an overview of the core of the cooperative games. A mathematical model for modeling cooperation in the power electric system is presented in Section 3. In the Section 4, we propose a case study of the process of fuel supply and electric power production in the Wassit thermal station using the core in the cooperative games. Section 5 introduces a comparative analysis of the results. Finally, we provide the conclusions in section 6.

2. An Overview of the Core in the Cooperative Games

Let $N = \{1, \dots, n\}$ be a universal set of players, the notation of cooperative game is specified by a pair (N, v), where v is a function of coalition value. For each coalition, $\emptyset \neq S \subseteq N$, a function $v(S)$ gaves the profits that any group of players can obtain, which is called the characteristic function of the game. Note that the characteristic function of the empty Ø set is $v\,(\emptyset) = 0$.

In the context of game theory, the definition of core value can be utilized as a solution concept of the game and provides a collaborative structure that balances the power of cooperation in cooperative game problems. The core in a cooperative game refers to a set of payoff distributions among players (participants in the game) such that no subset of players (called a coalition) would benefit by breaking away from the grand coalition (the entire group) and forming a smaller coalition.

In simpler terms, an allocation (or distribution of the total profit) is at the core if every coalition of players is at least as well off sticking with the grand coalition as they would be on their own. The following definition gives the core value $(Core\ (N, v))$ in the cooperative game.

Definition 1 [22]. Consider a cooperative game with a set of players $N = \{1,2, \dots, n\}$ and a characteristic function $v : 2^{\mathrm{N}} \rightarrow R$ that assigns a value $v(S)$ to each coalition $\emptyset \neq S \subseteq N$. The core $Core\ (N, v)$ is the set of all payoff distributions $(x_1, x_2, \dots, x_n)$ satisfying:

1. Efficiency: The total payoff is distributed among all players

$$\sum_{i=1}^{n} x_i = v(N)$$

2. Coalitional Rationality: No coalition $S \subseteq N$ can achieve a payoff greater than the sum of payoffs allocated to its members:

$$\sum_{i \in C} x_i \geq v(S) \ \ \forall\, S \subseteq N.$$

That is,

$$Core\ (N, v) = \left\{ (x_1, \dots, x_n) \in R^n : \sum_{i=1}^{n} x_i = v(N),\ \ \sum_{i \in C} x_i \geq v(S)\ \ \forall\, S\ \subseteq N \right\} \qquad (1)$$

* The Wassit thermal station is located in the Al-Zubaidiyah district, Wassit city of Iraq.

Example 1: Consider a scenario where three companies (A, B, and C) are oil producers. They can cooperate to maximize their total profit from oil production, but each company also has the option to produce and sell oil independently or form smaller coalitions.

The coalition values:

- Let $(A) = 30$, $v(B) = 40$, and $v(C) = 20$. These values represent the profits if each company works alone.
- If companies A and B form a coalition, they can achieve a profit of $v(A, B) = 90$ because of efficiencies and shared resources.
- If companies B and C form a coalition, they can achieve $v(B, C) = 80$.
- If companies A and C form a coalition, they can achieve $v(A, C) = 60$.
- If all three companies cooperate the total profit is $v(A, B, C) = 120$

The core calculation:

- The total profit $v(A, B, C) = 120$ must be divided among companies A, B, and C.
- The distribution (x_A , x_B, x_C) of the total profit must satisfy:

1. Efficiency: $x_A + x_B + x_C = 120$.
2. Coalitional Rationality:

- $x_A \geq 30$ (A would not accept less than what it could earn alone).
- $x_B \geq 40$ (B would not accept less than what it could earn alone
- $x_C \geq 20$ (C would not accept less than what it could earn alone).
- $x_A + x_B \geq 90$ (A and B together want at least as much as they earn as a coalition).
- $x_B + x_C \geq 80$ (B and C together want at least as much as they could earn as a coalition).
- $x_A + x_C \geq 60$ (A and C together want at least as much as they could earn as a coalition).

Therefore, the core of this cooperative game:

One possible allocation in the core could be x_A= 30, $x_B = 60$, and $x_C = 30$. This satisfies efficiency and coalitional rationality.

No subset of companies can do better in this distribution by breaking away from the grand coalition, making it stable and part of the core. If the distribution were outside the core, some companies would be incentivized to form a smaller coalition and earn more than they do in the grand coalition.

Therefore, the core represents the set of feasible allocations (or distributions of total payoff) among players that cannot be improved upon by any coalition of players.

3. A Mathematical Model for Electric Power Systems

This section describes a mathematical model of the cooperative game problem for modeling the general conflict among fuel supply producers and electric power stations using a core value-based optimization algorithm. Game theory is based on mathematical models of interactions between players or firms in a typical conflict. In our model, the common conflict between fuel supply producers and electric power stations can be formalized as a cooperative game with characteristic function v(S), where fuel supply producers are considered players, and their profit is the payoff.

Let $M = \{1, \dots, m\}$ be a set of electrical power stations, $E_1^c, \dots, E_m^c$ denote their capacities and $E_1^g, \dots, E_m^g$ denote their gate charges. The matrix defines the transportation costs $[c_{i,j}^t]$, where $c_{i,j}^t$ denotes the cost of transportation from the point of origin to the station (i.e., from the fuel supply producer $i \in N$ to the electric power station $j \in M$).

The amount of fuel sent by the producer $i \in N$ to the electric power station $j \in M$ in barrels is denoted by $x_{i,j}$. The set of fuel supply producers is $N = \{1, \dots, n\}$, and their supply productions are $E_1^p, \dots, E_n^p$.

The characteristic of the characteristic function $v(S)$, $S \subseteq N$ is given by the following form as an optimization model (adapted from [12]).

The optimization model of the characteristic function $v(S)$

$$v(S) = \min_{x_{i,j}:\, i \in S, j \in M} \sum_{j \in M} \sum_{i \in S} \left(c_{i,j}^t + c_j^p\right) x_{i,j} \quad (2)$$

Subject to

$$\sum_{i \in S} x_{i,j} \leq E_j^c - \sum_{i \in N \setminus S} x'_{i,j}, \quad \forall j \in M,$$
$$\sum_{j \in M} x_{i,j} = E_i^p, \quad \forall i \in S$$
$$x_{i,j} \geq 0, \quad \forall\, i \in S, \; j \in M,$$

where

$$\left\{x'_{i,j} : i \in N \setminus S, \forall\; j \in M\right\} = \min_{x_{i,j}:\, i \in N \setminus S, j \in M} \sum_{j \in M} \sum_{i \in N \setminus S} \left(c_{i,j}^t + c_j^p\right) x_{i,j}$$

and

$$\sum_{i \in N \setminus S} x_{i,j} \leq E_j^c, \quad \forall j \in M,$$
$$\sum_{j \in M} x_{i,j} = E_i^p, \quad \forall i \in N \setminus S,$$
$$x_{i,j} \geq 0, \quad \forall\, i \in N \setminus S, \; j \in M.$$

The fuel handling costs are calculated from the sum of the transportation costs and gate fees multiplied by the amount of fuel handled. The value of the characteristic function v(S) corresponds to the minimum total costs of the coalition members who made the correct decision after a coalition of all outsiders minimized the total cost of the operation. It provides an estimate of the cost of the case scenario in the conflict of setting up the power plant and fuel supply producers.

Consequently, the characteristic function will serve as the basis for all classes of games evaluated. In order to achieve a fair allocation of the cooperative game problem to the power plant and fuel supply producers, the resulting costs are compared using the Core values explained in Section 2 with a written algorithm and computer program (Python program) as a practical procedure. The process of computing the values of the characteristic function and the Core value is outlined in Algorithm 1.

Algorithm 1: The calculations of the characteristic functions and the core values.

Step1: For $\boldsymbol{N = \{1, \dots, n\}}$, $\boldsymbol{M = \{1, \dots, m\}}$**,**

Input: the costs of transportation fuel $(\boldsymbol{c^t_{1,1}}, \dots, \boldsymbol{c^t_{n,m}})$, and Pipelines transfer fees $\left(\boldsymbol{c^P_1}, \dots, \boldsymbol{c^P_m}\right)$,

Step2:

$for \quad i \in S, \quad S \subseteq N$

$for\ all\ \ j = 1\ to\ m$ do

Input: $x_{i,j}$;

Step3: if

$$x'_{i,j} = \underset{x_{i,j}:\, i\in N\setminus S, j\in M}{\text{Min}} \sum_{j\in M} \sum_{i\in N\setminus S} \left(c^t_{i,j} + c^p_j\right) x_{i,j} \ ;$$

end if

$end\ i$

$end\ j$

Step4: $for \quad i \in S,$

Find $v(S)$ via Eq. (2);

end for

Step5: Return Step2

Step6: $for \quad i \in S,$

Compute the Core via Eq. (1);

end for

Step7: Return Step 6**.**

4. Case Study: Fuel & Power Optimization at Wassit Thermal Station

This section applies the previously introduced optimization problem to analyze the fuel supply and electric power generation processes at the Wassit Thermal Station (Fig. 1). Located in the Al-Zubaidiyah district of Wassit, this facility is among Iraq's largest power stations, designed to produce 2540 MW per day over a 24-hour operation. The station comprises six oil (gas) fired units—four of 330 MW and two of 610 MW—totaling an installed capacity of 2540 MW. Developed by the Ministry of Electricity, the project is Iraq's largest thermal power plant, generating 18 billion kWh annually from its six units, which accounts for nearly 20% of the national power output and significantly alleviates the country's electricity supply challenges..

Figure 1. The Wassit thermal station.

This section examines the optimization problem for the processes at the Wassit Thermal Station (Fig. 1). Data on fuel supply and electric power production—including station capacity, gate fees, and transportation costs—were sourced from the station (see Table 1).

Table 1. The data on fuel supply and electric power production in Wassit thermal station.

W_i^p	C_{ij}^t	C_j^g	W_j^c
$W_1^p = 31447$BBL	$C_{11}^t = 2250$	$C_1^g = 1000$	$W_1^c = 2540MW$
$W_2^p = 44025$BBL	$C_{12}^t = 2450$	$C_1^g = 1000$	$W_1^c = 2540MW$
$W_3^p = 50315$BBL	$C_{13}^t = 2500$	$C_1^g = 1000$	$W_1^c = 2540MW$
$W_4^p = 44025$BBL	$C_{14}^t = 2200$	$C_1^g = 1000$	$W_1^c = 2540MW$

In the Wassit thermal station, there are four the fuel supply producers: The first is the Al-Ahdab oil field (AOF), the second is Badra oil field (BOF), the third is Al-Gharraf oil field (GOF), and the fourth is East Nahrawan oil field (NOF). Therefore, the set of producers is N= {1,2,3,4}, and there is only one station, which is the Wassit thermal station (i.e. M={1}).

The characteristic functions for all coalitions of the fuel supply producers, $S \subseteq N$={1,2,3,4} are computed by the optimization problem as mentioned above in Section 3, as shown in Table 2.

Table 2. The characteristic value for the fuel supply producers.

Coalitional set of oil fields	**{∅}**	**{1}**	**{2}**	**{3}**
(N, v)	0	102.2027	151.8862	176.1025
Coalitional set of oil fields	{4}	{1,2}	{1,3}	{1,4}
(N, v)	140.88	254.089	278.3052	243.0827
Coalitional set of oil fields	{2,3}	{2,4}	{3,4}	{1,2,3}
(N, v)	327.9887	292.7662	316.9825	430.1915
Coalitional set of oil fields	{1,2,4}	{1,3,4}	{2,3,4}	{1,2,3,4}
(N, v)	394.969	419.1832	468.8687	571.0715

By using the values in Table 2 and entering them into algorithm 1 of Section 3, the Core value for each fuel supply producer is shown in Table 3 and illustrated in Figure 2.

Table 3: The Core values of fuel supply producers.

Producer	$i = 1$(AOF)	$i = 2$(BOF)	$i = 3$(GOF)	$i = 4$(NOF)
$Core(N, v)$	102.203	151.887	176.103	140.88

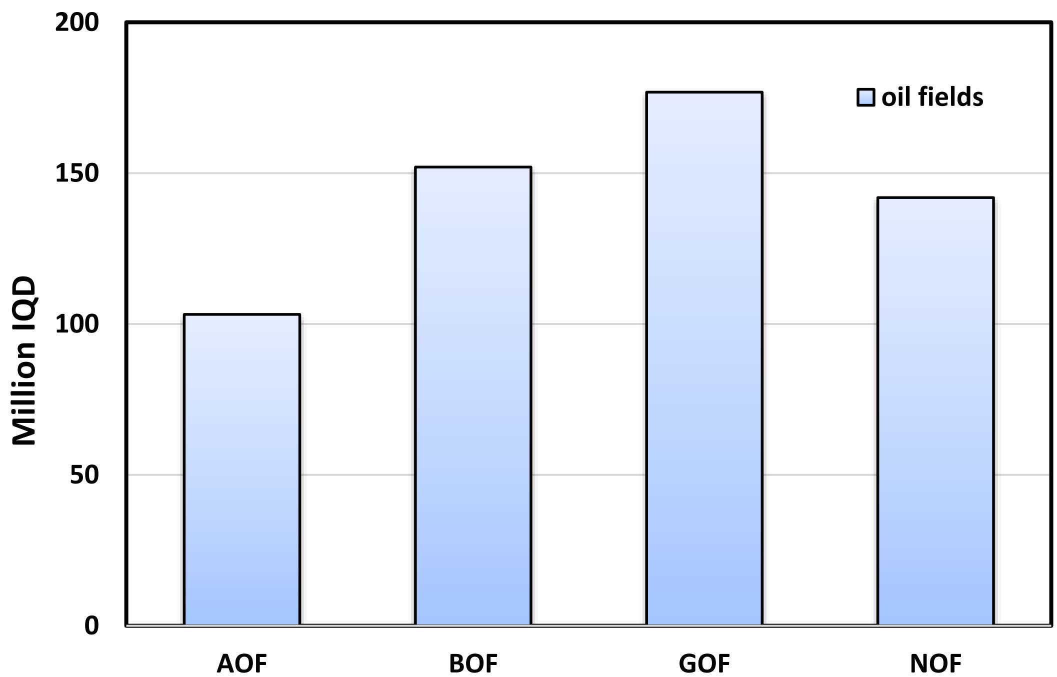

Figure 2. The core values of fuel supply producers.

5. Comparative Analysis of Results

To demonstrate the efficiency of the suggested model for enhancing fuel supplies and electricity generation at the Wasit Thermal Power Station, we compared the fuel suppliers at the station based on their preference relations and financial stability. Our analysis ensured that the distributions enable any group of stakeholders to enhance their position. Table 3 displays a comprehensive list of the fuel providers for the Wasit Thermal Power Station using the core value of each fuel supply producer.

We can compare fuel supply producers at Wassit thermal station through the preference relationship $\succ$. According to the preference relationship $\succ$, the four fuel supply producers will be ranked according to their contribution to the Wassit thermal station as shown in the Table 4. We conclude that the third fuel supply producer (i=3(GOF)) has the greatest ability to influence the outcome of the game and its contribution to the Wasit Thermal Power Station will be greater than the rest of the fuel supply producers.

Table 4: The comparative analysis of fuel supply producers through the preference relationship $\succ$.

Producer	i=3(GOF)	$\succ\ \boldsymbol{i=2(BOF)}$	$\succ \boldsymbol{i=4(NOF)}$	$\succ \boldsymbol{i=1(AOF)}$
$\boldsymbol{Core(N,v)}$	176.103	$\succ$151.887	$\succ$ 140.88	$\succ$ 102.203

Following the presentation on the significance of core and its connection to fuel goods, the primary concept of core became evident since there is no economic viability or financial gain from the departure of shareholders as a result of achieving stability and maximizing profit exiting the cooperative formula lacks stability.

Upon comparing the values of v(s) with those obtained from the core, it became apparent that there is no advantage or gain for any participant to leave the cooperative game. This conclusion is based on the observation that the core values are equal to or less than the values of v(s) when the basic condition is satisfied.

Validated on the proposed model's results generated by the survey research and analyzed during the current study, they are benchmarked and compared with available real-world datasets from Wassit thermal station.

Therefore, this study found that all fuel supply producers are engaged to different extents, indicating that each fuel product's viability and attainment of maximum profitability contribute to its stability within the group. As a result, this study will greatly help professionals to formulate and improve well-structured strategies for future electrical energy systems.

6. Conclusions

This paper utilizes a basic mathematical model that applies cooperative game theory to modeling cooperation among power stations and fuel supply producers. As per this model, we applied the methodology to study the case for the fuel supply and electric power production in the Wassit thermal station in case of cooperation. The initial outcomes demonstrate the utilization of the core values to illustrate the daily expenditure of an electric power plant in a cooperative setting, where electrical energy is generated by the collaborative exportation of fuel supply producers from oil fields. The outcomes of the proposed methodology will greatly help professionals to formulate and improve well-structured strategies for future electrical energy systems**.** For future research, our model can be extended to apply in similar contexts or how sensitive our approach is to changes in parameters like fuel costs, demand fluctuations, or regulatory policies. Furthermore, this approach can also address optimization problems in other sectors within production petrochemicals and more complicated optimization problems.

Conflict of Interest

The authors declare that they have no conflict of interest.

References

[1] M. Hemmati, M. Abapour, B. Mohammadi-Ivatloo, and A. Anvari-Moghaddam, "Optimal operation of integrated electrical and natural gas networks with a focus on distributed energy hub systems," *Sustainability*, vol. 12, no. 20, p. 8320, 2020.

[2] S. A. Mansouri, A. Ahmarinejad, M. Ansarian, M. S. Javadi, and J. P. S. Catalao, "Stochastic planning and operation of energy hubs considering demand response programs using Benders decomposition approach," *Int. J. Electr. Power Energy Syst.*, vol. 120, p. 106030, 2020.

[3] M. S. Javadi, M. Lotfi, A. E. Nezhad, A. Anvari-Moghaddam, J. M. Guerrero, and J. P. S. Catalão, "Optimal operation of energy hubs considering uncertainties and different time resolutions," *IEEE Trans. Ind. Appl.*, vol. 56, no. 5, pp. 5543–5552, 2020.

[4] M. Nikzad and A. Samimi, "Integration of designing price-based demand response models into a stochastic bi-level scheduling of multiple energy carrier microgrids considering energy storage systems," *Appl. Energy*, vol. 282, p. 116163, 2021.

[5] M. Hamidi, H. Liao, and F. Szidarovszky, "Non-cooperative and cooperative game-theoretic models

for usage-based lease contracts," *Eur. J. Oper. Res.*, vol. 255, no. 1, pp. 163–174, 2016.

[6] B. Lokeshgupta and S. Sivasubramani, "Cooperative game theory approach for multi-objective home energy management with renewable energy integration," *IET Smart Grid*, vol. 2, no. 1, pp. 34–41, 2019.

[7] H. Hadi and J. Abbas, "Studying well-organized strategies for the waste management system in Baghdad using cooperative game theory," in *AIP Conference Proceedings*, AIP Publishing, 2023.

[8] E. H. Ouda, S. H. Khazaal, and J. Abbas, "An Application of Cooperative Game Theory in Oil Refining Sites: Case Study of Dora Refinery in Iraq," in *International Conference on Intelligent and Fuzzy Systems*, Springer, 2023, pp. 592–599.

[9] L. Han, "A scalable cooperative game theoretic framework for coalitional energy management," 2020, *University of Oxford*.

[10] H. Hadi and J. Abbas, "A Comparison of Results among Waste Management Producers: A Case Study for the Process of Waste Management," *Eng. Technol. J.*, vol. 41, no. 04, pp. 586–591, 2023.

[11] L. Skovsgaard and I. G. Jensen, "Recent trends in biogas value chains explained using cooperative game theory," *Energy Econ.*, vol. 74, pp. 503–522, 2018.

[12] H. Hadi and J. Abbas, "An Application of the Banzhaf Values for Cooperating Among Producers of Waste Processing in the Al-Mahmudiya Factory," *Baghdad Sci. J.*, vol. 20, no. 4, p. 1359, 2023.

[13] A. B. da Silva Rocha and G. M. Salomão, "Environmental policy regulation and corporate compliance in evolutionary game models with well-mixed and structured populations," *Eur. J. Oper. Res.*, vol. 279, no. 2, pp. 486–501, 2019.

[14] J. Abbas, "The Banzhaf interaction index for bi-cooperative games," *Int. J. Gen. Syst.*, vol. 50, no. 5, pp. 486–500, 2021.

[15] R. I. Sabri, M. N. Mohammedali, and J. Abbas, "An application of non-additive measures and corresponding integrals in tourism management," *Baghdad Sci. J.*, vol. 17, no. 1, pp. 172–177, 2020.

[16] J. A. Ghafil, "A new approach to the bipolar Shilkret integral," *Math. Comput. Sci.*, vol. 3, no. 4, pp. 46–54, 2022.

[17] J. Abbas, "The balancing bipolar Choquet integrals," *Int. J. Innov. Comput. Inf. Control*, vol. 17, no. 3, pp. 949–957, 2021.

[18] J. Abbas, "The bipolar Choquet integrals based on ternary-element sets," *J. Artif. Intell. Soft Comput. Res.*, vol. 6, no. 1, pp. 13–21, 2016.

[19] M. Hesham and J. Abbas, "Multi-criteria decision making on the best drug for rheumatoid arthritis," *Iraqi J. Sci.*, pp. 1659–1665, 2021.

[20] J. Abbas, "The 2-additive choquet integral of bi-capacities," in *International Conference on Artificial Intelligence and Soft Computing*, Springer, 2019, pp. 287–295.

[21] A. Naeem and J. Abbas, "A computational model for multi-criteria decision making in traffic JAM problem," *J. Autom. Mob. Robot. Intell. Syst.*, vol. 15, 2021.

[22] Y. Kannai, "The core and balancedness," *Handb. game theory with Econ. Appl.*, vol. 1, pp. 355–395, 1992.